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Causality violation and naked time machines in AdS_5

Marco M. Caldarelli
Dipartimento di Fisica dell'Università di Trento and
INFN, Gruppo Collegato di Trento
Via Sommarive 14, 38050 Povo (TN), Italy
E-mail: caldarel@science.unitn.it

Dietmar Klemm
Dipartimento di Fisica dell'Università di Milano and
INFN, Sezione di Milano
Via Celoria 16, 20133 Milano, Italy
E-mail: dietmar.klemm@mi.infn.it

Wafic A. Sabra
Center for Advanced Mathematical Sciences (CAMS) and
Physics Department, American University of Beirut, Lebanon
E-mail: ws00@aub.edu.lb

Abstract: We study supersymmetric charged rotating black holes in AdS_5, and show that closed timelike curves occur outside the event horizon. Also upon lifting to rotating D3 brane solutions of type-IIB supergravity in ten dimensions, closed timelike curves are still present. We believe that these causal anomalies correspond to loss of unitarity in the dual $\mathcal{N} = 4$, $D = 4$ super Yang-Mills theory, i.e. the chronology protection conjecture in the AdS bulk is related to unitarity bounds in the boundary CFT. We show that no charged or uncharged geodesic can penetrate the horizon, so that the exterior region is geodesically complete. These results still hold true in the quantum case, i. e. the total absorption cross section for Klein-Gordon scalars propagating in the black hole background is zero. This suggests that the effective temperature is zero instead of assuming the naively found imaginary value.

Keywords: Black Holes in String Theory, AdS-CFT Correspondance, Black Holes, Supergravity Models.

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Contents

1. Introduction

It is well-known that supersymmetry does not exclude the existence of closed timelike curves. Simple examples are flat space with periodically identified time and anti-de Sitter space. However, these spacetimes are not simply connected, and one can avoid CTCs by passing to the universal covering. One might therefore assume that in simply connected supersymmetric spaces closed timelike curves do not exist. This however turned out not to be the case, for example the BMPV black hole [1, 2], which is a BPS solution of $D = 5$, $\mathcal{N} = 2$ supergravity[1] with trivial fundamental group, admits CTCs. These black holes are characterized by a charge parameter

[1] Cf. also [3] for a generalization of the BMPV solution to the case of $D = 5$, $\mathcal{N} = 2$ supergravity coupled to vector multiplets.

q and a rotation parameter a. For $a^2 < q^3$ (hereafter referred to as the "under-rotating" case), CTCs occur only in the region beyond the horizon, and thus an external observer cannot use them to construct a time machine. This is quite similar to the case of the Kerr black hole. However, in the "over-rotating" case $a^2 > q^3$, CTCs are present in the exterior region outside the event horizon [4]. This happens in spite of supersymmetry and the presence of a matter stress tensor satisfying the dominant energy condition. Various aspects of these naked time machines have been extensively analyzed in [5, 6]. Among other things, the authors of [5] showed that causal geodesics cannot penetrate the horizon for the over-rotating solution (repulson-like behaviour), and so the exterior region is geodesically complete (with respect to causal geodesics). Furthermore, in [6] it was shown that upon lifting of the over-rotating BMPV black hole to a solution of type-IIB string theory and passing to the universal covering space, the causal anomalies disappear. The rotating BMPV black hole, which corresponds to a D1-D5-Brinkmann wave system in type-IIB string theory [6], admits a dual description in terms of an $\mathcal{N} = 4$ two-dimensional superconformal field theory. The causality bound $a^2 = q^3$ is then equivalent to the unitarity bound in this CFT [6], obtained by requiring unitary representations of the superconformal algebra, which implies an inequality involving the central charge, conformal weight and R-charge.

Recently, the generalization of the BMPV solution to the case of $D = 5$, $\mathcal{N} = 2$ gauged supergravity has been found in [7, 8]. These black holes, which also preserve four supercharges, were shown to suffer from CTCs outside the horizon for all parameter values, as soon as rotation is turned on. Due to these causal anomalies, the naively computed Hawking temperature and the Bekenstein-Hawking entropy are imaginary. According to the AdS/CFT correspondence [9], these black hole solutions, which asymptotically approach anti-de Sitter space, should be dual to $\mathcal{N} = 4$, $D = 4$ super Yang-Mills theory in the presence of R-charges. Similar to the ungauged case, where the appearance of CTCs was shown to be related to unitarity violation in a two-dimensional CFT, we thus expect that the causal anomalies occurring in the charged rotating AdS black holes correspond to loss of unitarity in $\mathcal{N} = 4$, $D = 4$ super Yang-Mills theory. One has thus a relation between macroscopic causality in the AdS bulk and microscopic unitarity in the boundary CFT.

In this paper, we will be concerned with a detailed study of the rotating AdS black holes found in [7, 8] and their causal anomalies. Our work is organized as follows. In sections 2 and 3, we review the solution and analyze its geometric properties. In section 4 the throat geometry describing the near-horizon limit of the extremal solutions is determined. In 5 it is shown that no charged or uncharged geodesic can penetrate the horizon, implying that the exterior region is geodesically complete. This behaviour still holds true in the quantum case, that is the absorption cross section of the hole for Klein-Gordon scalars is zero. This suggests that the effective Hawking temperature is zero instead of assuming the naively found imaginary value.

The results obtained in section 5 are thus very similar to those of the ungauged case that was studied in [5, 6]. In 6 our results are generalized to the case when the gauged supergravity theory is coupled to vector multiplets. The solution of the $STU = 1$ model is lifted to a solution of type-IIB supergravity in ten dimensions in section 7, yielding a rotating D3 brane wrapping $\mathcal{S}^3$. It is then shown that the original CTCs present in five dimensions disappear upon lifting, but new CTCs show up. Therefore, in contrast to the BMPV black holes, we have no resolution of causal anomalies in higher dimensions. We conclude in 8 with some final remarks.

2. The black hole solution

We consider Einstein-Maxwell theory in five dimensions with a negative cosmological constant and a Chern-Simons term for the abelian gauge field. The action is given by

$$S=\frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left(R-2\Lambda-\frac{1}{12}F_{\mu\nu}F^{\mu\nu}\right)+\frac{1}{16\pi G_5}\int d^5x\frac{1}{108}\epsilon^{\mu\nu\rho\sigma\lambda}F_{\mu\nu}F_{\rho\sigma}A_\lambda\,, \tag{2.1}$$

where R is the scalar curvature, $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ the abelian field-strength tensor, G_5 denotes the five-dimensional Newton constant and $\Lambda=-6g^2$ the cosmological constant. (2.1) is the bosonic truncation of the gauged five-dimensional $\mathcal{N}=2$ pure supergravity theory.

In [7], a charged rotating solution of this theory was found. Its metric reads

$$ds^2=-g^2r^2\,dt^2-\frac{1}{r^4}\left[\left(r^2-q\right)dt-a\sin^2\theta\,d\phi+a\cos^2\theta\,d\psi\right]^2+\frac{dr^2}{V(r)}+r^2\,d\Omega_3^2, \tag{2.2}$$

where

$$V(r)=\left(1-\frac{q}{r^2}\right)^2+g^2r^2-\frac{g^2a^2}{r^4}, \tag{2.3}$$

and the gauge fields are given by

$$A_\phi=-\frac{3a}{r^2}\sin^2\theta,\qquad A_\psi=\frac{3a}{r^2}\cos^2\theta,\qquad A_t=3\left(1-\frac{q}{r^2}\right). \tag{2.4}$$

Here, $d\Omega_3^2$ denotes the standard metric on the unit three-sphere,

$$d\Omega_3^2=d\theta^2+\sin^2\theta\,d\phi^2+\cos^2\theta\,d\psi^2\,, \tag{2.5}$$

with the angles θ, ϕ and ψ parametrizing $\mathcal{S}^3$ ranging in $\theta\in[0,\pi/2]$, $\phi\in[0,2\pi[$, $\psi\in[0,2\pi[$. For convenience, we define two other functions, which will be of use in the following, namely

$$\Delta(r)\equiv 1-\frac{q}{r^2}\,,\qquad \Delta_L(r)\equiv 1-\frac{a^2}{r^6}\,. \tag{2.6}$$

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Using the diffeomorphism $\mathcal{S}^3 \cong \mathrm{SU}(2)$, one can parametrize the three-sphere with the Euler parameters (α, β, γ) of SU(2), related to the angular variables θ, ϕ and ψ through the transformation

$$\alpha = \psi + \phi\,, \quad \beta = 2\theta\,, \quad \gamma = \psi - \phi\,. \tag{2.7}$$

In these coordinates, the metric of the three-sphere takes the form

$$d\Omega_3^2 = \frac{1}{4}\left(d\alpha^2 + d\beta^2 + d\gamma^2 + 2\cos\beta\; d\alpha\; d\gamma\right)\,, \tag{2.8}$$

with $\alpha \in [0, 2\pi[$, $\beta \in [0, \pi]$ and $\gamma \in [0, 4\pi[$. The isometry group of the three-sphere $\mathrm{SO}(4) \cong \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ consists of two copies of the SU(2) group. The left-invariant vector fields ξ^R generating the right translations are

$$\begin{aligned}
\xi_1^R &= \cos\gamma\,\mathrm{cosec}\,\beta\;\partial_\alpha - \sin\gamma\;\partial_\beta - \cos\gamma\cot\beta\;\partial_\gamma\,,\\
\xi_2^R &= \sin\gamma\,\mathrm{cosec}\,\beta\;\partial_\alpha + \cos\gamma\;\partial_\beta - \sin\gamma\cot\beta\;\partial_\gamma\,,\\
\xi_3^R &= \partial_\gamma\,,
\end{aligned} \tag{2.9}$$

whereas the right-invariant vector fields generating the left translations read

$$\begin{aligned}
\xi_1^L &= -\sin\alpha\cot\beta\;\partial_\alpha + \cos\alpha\;\partial_\beta + \sin\alpha\,\mathrm{cosec}\,\beta\;\partial_\gamma\,,\\
\xi_2^L &= -\cos\alpha\cot\beta\;\partial_\alpha - \sin\alpha\;\partial_\beta + \cos\alpha\,\mathrm{cosec}\,\beta\;\partial_\gamma\,,\\
\xi_3^L &= \partial_\alpha\,.
\end{aligned} \tag{2.10}$$

They satisfy the commutation relations

$$\left[\xi_a^R, \xi_b^R\right] = -\epsilon_{abc}\xi_c^R\,, \qquad \left[\xi_a^L, \xi_b^L\right] = \epsilon_{abc}\xi_c^L\,, \qquad \left[\xi_a^R, \xi_b^L\right] = 0\,. \tag{2.11}$$

Introducing the left-invariant one-forms σ_i ($i = 1, 2, 3$), dual to the left-invariant vector fields ξ_a^R in the sense that $(\xi_a^R, \sigma_b) = \delta_{ab}$,

$$\begin{aligned}
\sigma_1 &= -\sin\gamma\; d\beta + \cos\gamma\sin\beta\; d\alpha\,,\\
\sigma_2 &= \cos\gamma\; d\beta + \sin\gamma\sin\beta\; d\alpha\,,\\
\sigma_3 &= d\gamma + \cos\beta\; d\alpha\,,
\end{aligned} \tag{2.12}$$

the metric of the three sphere can be written as

$$d\Omega_3^2 = \frac{1}{4}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2\right)\,. \tag{2.13}$$

The left-invariant one-forms satisfy $d\sigma_1 = \sigma_2 \wedge \sigma_3$ (together with its cyclic permutations). Rewriting the black hole metric (2.2) in terms of the one-forms σ^i, we obtain

$$ds^2 = -g^2r^2\; dt^2 - \Delta^2(r)\left(dt + \frac{a}{2r^2\Delta(r)}\;\sigma_3\right)^2 + \frac{dr^2}{V(r)} + \frac{r^2}{4}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2\right)\,, \tag{2.14}$$

and the gauge-field 1-form is given by

$$A = 3\Delta(r)\; dt + \frac{3a}{2r^2}\;\sigma_3\,. \tag{2.15}$$

3. Geometric properties of the black hole spacetime

3.1 "Horizons"

The horizons of the black hole are located at the zeroes of the function $V(r)$. Using the dimensionless parameters $\alpha = g^3 a$, $\rho = g^2 q$ and $\zeta = gr$, the problem reduces to finding the zeroes of the polynomial $f(\zeta)$ defined by

$$f(\zeta) = \zeta^6 + \zeta^4 - 2\rho\zeta^2 + \rho^2 - \alpha^2\,. \tag{3.1}$$

If $\rho \leq 0$, we have $f'(\zeta) = 0$ only for $\zeta = 0$, and hence $f(\zeta)$ is an increasing function of its variable. As $f(0) = \rho^2 - \alpha^2$, for $|\rho| > |\alpha|$ we have a naked singularity, for $|\rho| = |\alpha|$ a single horizon at $\zeta_+ = 0$, and for $|\rho| < |\alpha|$ a single horizon located at some point $\zeta_+ > 0$, with a spacelike singularity.

The situation is more interesting for $\rho > 0$. In this case, the derivative f' has an additional positive root for

$$\zeta^2 = \bar{\zeta}^2 \equiv \frac{1}{3}\left(\sqrt{1+6\rho} - 1\right). \tag{3.2}$$

Hence, if $\bar{\zeta}$ is also a root of f, it is a double root and thus the black hole is extremal. This occurs for $\alpha^2 = \alpha^2_{extr} \equiv \bar{\zeta}^6 + \bar{\zeta}^4 - 2\rho\bar{\zeta}^2 + \rho^2$. The critical rotation parameter is given by

$$\alpha^2_{extr}(\rho) = \rho^2 + \frac{2}{3}\rho + \frac{2}{27} - \frac{2}{27}\left(1+6\rho\right)^{3/2}\,. \tag{3.3}$$

It is easy to verify that $\rho^2 > \alpha^2_{extr}$ holds for $\rho > 0$. Hence, for positive ρ, we have the following cases:

- $\rho^2 > \alpha^2_{\rm extr}(\rho) > \alpha^2$: There is no positive root, and hence no horizon and therefore we are left with a naked singularity.
- $\alpha^2 = \alpha^2_{\rm extr}(\rho)$: There is a double root $\zeta_+ > 0$ of f, and thus the black hole is extremal. The singularity is timelike, and the horizon is exactly at $\zeta_+ = \bar{\zeta}$ given by equation (3.2).
- $\rho^2 > \alpha^2 > \alpha^2_{\rm extr}(\rho)$: There are two roots ζ_+ and ζ_-, corresponding to an outer event horizon at ζ_+, and an inner Cauchy horizon at ζ_-. In this case, the singularity is spacelike.
- $\alpha^2 = \rho^2$: The Cauchy horizon collapses in the singularity, $\zeta_- = 0$, leaving a simple root $\zeta_+ > 0$ corresponding to the bifurcated horizon of the black hole.
- $\alpha^2 > \rho^2$: There is a single positive root ζ_+, and the metric describes a black hole with a spacelike singularity hidden by a bifurcated horizon located at ζ_+.

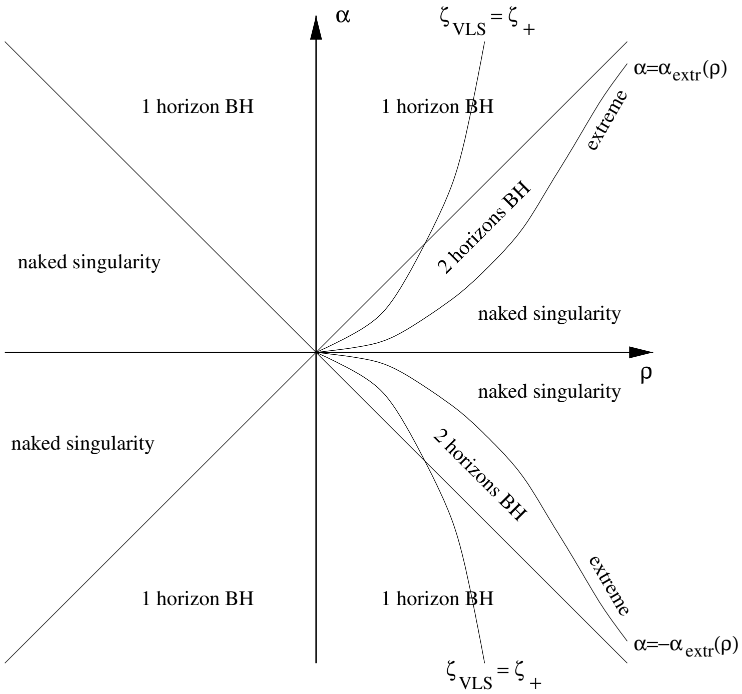

Figure 1: Properties of the metric on the (α, ρ)-plane.

The results are summarized in figure 1, where the properties of our metric are shown in the (α, ρ)-plane.

The one-parameter subfamily of extreme black holes can be parametrized by the location ζ_+ of the double root of f. In this case, f takes the form

$$f(\zeta) = \left(\zeta^2 - \zeta_+^2\right)^2 \left(\zeta^2 + 2\zeta_+^2 + 1\right), \tag{3.4}$$

and the rotation and charge parameters are given by

$$\rho = \frac{1}{2}\left(3\zeta_+^2 + 2\right)\zeta_+^2, \qquad \alpha^2 = \left(\frac{9}{4}\zeta_+^2 + 1\right)\zeta_+^6. \tag{3.5}$$

With these relations, it is possible to explicitly write the metric components of the extreme metrics as a function of ζ_+.

We finally look for ergospheres. Ergoregions occur when the norm of the asymptotically timelike Killing vector ∂_t becomes positive. But $(\partial_t, \partial_t) = -g^2r^2 - \Delta^2(r) < 0$ for every r, hence ∂_t is always timelike and there are no ergospheres in the manifold under consideration.

3.2 Velocity of light surfaces and time machines

It can be easily seen that our metric allows for closed timelike curves. For instance, the norm of ∂_γ is $r^2/4 - a^2/(4r^4)$ and becomes timelike for $r < a^{1/3}$. The integral curves of this Killing vector being closed circles, we see that for r sufficiently small there are CTCs. It is then essential to see whether these curves arise behind the horizon — as for the Kerr black hole — or if they are outside, thus yielding a naked time machine undermining causality.

On the boundary between the time machine region and the causal region, the Killing vector ∂_γ becomes lightlike. As in [5] we shall call this boundary the *velocity of light surface* (VLS); it is a timelike surface located in $\zeta_{VLS} = \alpha^{1/3}$. If this surface is not hidden by a horizon, our spacetime is a naked time machine.

Rearranging terms in the polynomial f, we obtain $f(\zeta) = \zeta^6 - \alpha^2 + (\zeta^2 - \rho)^2$, which implies

$$\zeta_{VLS} = |\alpha|^{1/3} \geq \zeta_+ \tag{3.6}$$

for every choice of the parameters (α, ρ) giving rise to a horizon. Hence, the velocity of light surface is always *outside* of the horizon, and the black hole (2.2) has naked CTCs for every choice of parameters[2] (this contrasts with its asymptotically flat ($g = 0$) counterpart, where the VLS is hidden by an event horizon for $a^2 < q^3$ [5]). When equality holds in equation (3.6), the VLS coincides with the horizon, $\zeta_{VLS} = \zeta_+$. This occurs for $f(\zeta_{VLS}) = (\alpha^{2/3} - \rho)^2 = 0$, that is for the parameters of the black hole satisfying $\alpha_{VLS}^2 = \rho^3$. This curve is shown in figure 1.

Analyzing the sign of $(\partial_t, \partial_\gamma)$, we see that ∂_γ is future pointing for $\alpha > 0$, and past pointing for $\alpha < 0$.

3.3 Symmetries and killing tensors

The spacetime (2.14) is stationary, hence ∂_t is a Killing vector. Turning to the isometries of the three-sphere (2.9), (2.10), we see that the rotation in the right SU(2) sector of SO(4) breaks the ξ_1^R and ξ_2^R right translations, and hence breaks the $\mathrm{SU}(2)_R$ isometry group to $\mathrm{U}(1)_R$ generated by ξ_3^R. On the other side, it can be verified that the left rotation isometry subgroup $\mathrm{SU}(2)_L$ remains unbroken, and the vectors ξ_i^L given in (2.10) are Killing vectors of the metric. Thus, the isometry group of our spacetime is $\mathbb{R} \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_R$.

In the following sections, we shall study the geodesics of this metric. The Killing vectors give rise to three conserved charges, which will show useful in the separation of the angular part of the equations. Furthermore, a fourth conserved quantity is provided by the Stäckel-Killing tensor

$$K^{\mu\nu} = \sum_{i=1}^{3} \xi_i^{L\mu} \xi_i^{L\nu} = \sum_{i=1}^{3} \xi_i^{R\mu} \xi_i^{R\nu} \,. \tag{3.7}$$

This is the Casimir invariant of any of the SU(2) subgroups of the SO(4) rotation group, which coincide in the scalar representation. As a consequence, breaking

[2]The appearance of naked CTCs is a general feature of spinning black holes that are solutions of the vacuum Einstein equations in any odd number of dimensions, when all the rotation parameters are nonvanishing, as shown in [10].

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$SU(2)_R \to U(1)_R$ does not decrease the actual number of constant of motions, and even in the rotating case the geodesic and wave equations remain completely separable. We refer the reader to [5] for a complete discussion.

4. Throat geometry of the near-extreme solution

To perform the near-extremal, near-horizon limit, it is useful to start from the canonical form of the metric

$$ds^2 = -\frac{V(r)}{\Delta_L}dt^2 + \frac{dr^2}{V(r)} + \frac{r^2}{4}\Delta_L\left(\sigma_3 - \omega\ dt\right)^2 + \frac{r^2}{4}\left(\sigma_1^2 + \sigma_2^2\right), \tag{4.1}$$

where we have defined

$$\omega = \frac{2a\Delta}{r^4\Delta_L}. \tag{4.2}$$

To obtain a non-singular near-horizon limit, we have to approach the extremal limit while moving towards the horizon. Following [14], we keep the charge q fixed and parametrize the rotation parameter as $a = a_{extr}(q)(1 + k\epsilon^2)$, where k is an arbitrary constant, and ϵ is the extremality parameter. Let us call r_e the location of the double root of $V(r)$ which develops for $\epsilon = 0$. For simplicity, we define also the rotation parameter at extremality $a_e = a_{extr}(q)$ and the angular velocity at extremality

$$\omega_e = \left.\frac{2a_e\Delta}{r^4\Delta_L}\right|_{extr} = \frac{4a_e}{3r_e^4}. \tag{4.3}$$

In order to take the near-horizon limit as we approach the extremal solution, we define the new coordinates (ψ, R, γ_c)

$$r = r_e + \epsilon R\,, \qquad \psi = \epsilon t\,, \qquad \gamma_c = \gamma - \omega_e t\,; \tag{4.4}$$

hence σ_3 becomes

$$\sigma_c = \sigma_3 - \omega_e\ dt\,. \tag{4.5}$$

One then finally performs the $\epsilon \to 0$ limit. The function $V(r)$ reads

$$V(r) = \left[4\left(1 + 3g^2r_e^2\right)\frac{R^2}{r_e^2} - \frac{2g^2a_{extr}^2(r_e)}{r_e^4}k\right]\epsilon^2 + \mathcal{O}\left(\epsilon^3\right) \tag{4.6}$$

and the near horizon metric is

$$ds^2 = \frac{4f(R)}{9g^2r_e^2}\ d\psi^2 + \frac{dR^2}{f(R)} - \frac{9g^2r_e^4}{16}\left(\sigma_c - \frac{16a_e}{9g^2r_e^7}\ Rd\psi\right)^2 + \frac{r_e^2}{4}\left(\sigma_1^2 + \sigma_2^2\right), \tag{4.7}$$

with

$$f(R) = 4\left(1 + 3g^2r_e^2\right)\frac{R^2}{r_e^2} - 2k\frac{g^2a_e^2}{r_e^4}. \tag{4.8}$$

Similar throat solutions (with $k = 0$) have been found in [15, 16] for Kerr black holes and in [17, 18] for black holes in five-dimensional dilaton-axion gravity. To obtain a finite limit for the vector potential, we have to perform a gauge transformation $A \mapsto A - 2dt$. The limit can then be performed safely, obtaining

$$A = \frac{2R}{r_e}\, d\psi + \frac{3a_e}{2r_e^2}\, \sigma_c\,. \tag{4.9}$$

For $k = 0$, the throat solution has an enhanced isometry group; the original $\mathrm{SU}(2)_L \times \mathrm{U}(1)_R$ symmetry generated by ξ_i^L and ξ_3^R is still present, but we have now three additional Killing vectors

$$\begin{aligned}
\chi_1 &= \frac{\partial}{\partial\psi}\,, \qquad\qquad \chi_2 = \psi\frac{\partial}{\partial\psi} - R\frac{\partial}{\partial R}\,, \\
\chi_3 &= \left[\frac{\psi^2}{2} - \frac{9g^2r_e^6}{128\,(1+3g^2r_e^2)^2}\frac{1}{R^2}\right]\frac{\partial}{\partial\psi} - \psi R\frac{\partial}{\partial R} - \frac{a_e}{4r_e\,(1+3g^2r_e^2)^2}\frac{1}{R}\frac{\partial}{\partial\gamma_c}\,,
\end{aligned}$$

which obey the commutation relations of the $\mathrm{SL}(2,\mathbb{R})$ algebra. As a consequence, the full isometry group of the throat solution is $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_R$. A non-vanishing k, in contrast, breaks the $\mathrm{SL}(2,\mathbb{R})$ symmetry group and only χ_1 remains a Killing vector. In this case the isometry group is $\mathbb{R} \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_R$, as in the full metric.

The near-horizon limit (4.7), (4.9) with $k = 0$ is a special case of a general class of solutions to (2.1), given by

$$\begin{aligned}
ds^2 &= c_1^2(d\chi^2 + \sinh^2\chi\; d\zeta^2) + c_2^2(d\beta^2 + \sin^2\beta\; d\alpha^2) - (d\gamma + b_1A_1 + b_2A_2)^2\,, \\
A &= a_1A_1 + a_2A_2\,, \qquad A_1 = \cosh\chi\; d\zeta\,, \qquad A_2 = \cos\beta\; d\alpha\,.
\end{aligned} \tag{4.10}$$

(Note that dA_1 and dA_2 are essentially the Kähler forms on H^2 and S^2 respectively). The constants a_i, b_i and c_i are subject to the constraints

$$\begin{aligned}
a_1^2 &= 6b_1^2 - 6c_1^2 + 3b_2^2(c_1/c_2)^4\,, \qquad a_2^2 = 6b_2^2 + 6c_2^2 + 3b_1^2(c_2/c_1)^4\,, \\
12g^2 &= c_1^{-2} - c_2^{-2}\,, \quad 2c_1^2c_2^2a_1a_2 + \sqrt{3}(c_2^4a_1b_1 + c_1^4a_2b_2) = 0\,,
\end{aligned}$$

following from the equations of motion of the action (2.1). This system leaves two parameters undetermined, e. g. we can choose c_1^2 and the ratio b_1/a_2 freely.[3] It is straightforward to show that the general solution (4.10) is a homogeneous manifold $[\mathrm{SO}(2,1) \times \mathrm{SU}(2) \times \mathrm{U}(1)]/[\mathrm{U}(1) \times \mathrm{U}(1)]$. It should therefore be interesting to study this spacetime in the context of holography and coset spaces [20].

[3]In particular, for the choice $a_1 = b_2 = 0$, $a_2 = 1/g$, $b_1^2 = c_1^2 = 1/(9g^2)$, $c_2^2 = -1/(3g^2)$, one obtains the solutions considered in [19], of the form $AdS_3 \times H^2$, where the AdS_3 part is written as an S^1 bundle over H^2. These solutions preserve half of the supersymmetries.

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5. Maximal extension of the black hole solution

5.1 Geodesics of the black hole solution

The Hamilton-Jacobi equation for the action function $S(x^\mu)$, describing the geodesics of the metric (2.2), reads

$$\begin{aligned} & -\frac{\Delta_L}{V}\left(\frac{\partial S}{\partial t}\right)^2 + V\left(\frac{\partial S}{\partial r}\right)^2 - \frac{4a\Delta}{r^4 V}\frac{\partial S}{\partial t}(L_3 S) + \\ & + \frac{4}{r^2}\left((L_1 S)^2 + (L_2 S)^2 + (L_3 S)^2\right) + \frac{4g^2a^2}{r^6 V}(L_3 S)^2 = -m^2 \,, \end{aligned} \tag{5.1}$$

where we have defined $L_i = \xi_i^R$. The AdS signature is completely encoded in the function V and the last term of the left hand side, proportional to g^2. This partial differential equation is completely integrable, thanks to the symmetries of the metric (2.2): ∂_t, ∂_α and ∂_γ are Killing vectors of the spacetime. This suggests the ansatz

$$S = -Et + H(\alpha,\beta,\gamma) + W(r)\,, \qquad H(\alpha,\beta,\gamma) = j_L\alpha + j_R\gamma + \chi(\beta)\,. \tag{5.2}$$

A further conserved quantity, j^2, arises from the Stäckel-Killing tensor of the spacetime,

$$(L_1 H)^2 + (L_2 H)^2 + (L_3 H)^2 = j^2\,, \tag{5.3}$$

which allows to determine χ,

$$\frac{\partial\chi}{\partial\beta} = \pm\sqrt{j^2 - \frac{1}{\sin^2\beta}\left(j_R^2 + j_L^2 - 2j_L j_R\cos\beta\right)}\,. \tag{5.4}$$

Using the ansatz for the action, the Hamilton-Jacobi equation reduces to

$$W'(r)^2 = \frac{1}{V^2(r)}\left(\Delta_L E^2 - V\left(m^2 + \frac{4j^2}{r^2}\right) - \frac{4\Delta}{r^4}aj_R E - \frac{4g^2a^2}{r^6}j_R^2\right), \tag{5.5}$$

which can be integrated.

The geodesic equations follow then from the action,

$$\frac{dx^\mu}{d\lambda} = -g^{\mu\nu}\frac{\partial S}{\partial x^\nu}\,, \tag{5.6}$$

where λ denotes the geodesic parameter. For the radial motion, we obtain

$$\left(\frac{dr}{d\lambda}\right)^2 = \Delta_L E^2 - V\left(m^2 + \frac{4j^2}{r^2}\right) - \frac{4\Delta}{r^4}aj_R E - \frac{4g^2a^2}{r^6}j_R^2\,. \tag{5.7}$$

One would like to know if geodesics can cross the velocity of light surface and how far they can travel towards the horizon. Obviously, the motion is allowed as long as the right hand side of equation (5.7) is positive. Let us consider first geodesics with $j_R = 0$. In this case we have

$$\dot{r}^2 = \Delta_L E^2 - V\left(m^2 + \frac{4j^2}{r^2}\right) < \Delta_L E^2\,, \tag{5.8}$$

because $V > 0$ for $r > r_+$. Moreover, Δ_L is positive outside the velocity of light surface, but becomes negative for $r < r_{VLS}$. Hence *particles with $j_R = 0$ cannot cross the VLS*. To enter the time machine, we have to lower the potential barrier by putting some angular momentum along the γ direction. For $j_R \neq 0$, we have two additional terms in (5.7): the last term is always negative, but the third term can become positive if $\Delta a j_R < 0$. Hence, the geodesics can enter the time machine only if they have spin *opposite* to the dragging effects of the spacetime.

Let us examine now if such a geodesic can cross the event horizon. For $r = r_+$, the r.h.s. of equation (5.7) becomes

$$\left(\frac{dr}{d\lambda}\right)^2\bigg|_{r_+} = \Delta_L^+ E^2 - \frac{4\Delta_+}{r_+^4} a j_R E - \frac{4g^2a^2}{r_+^6} j_R^2\,, \tag{5.9}$$

where Δ_L^+ and Δ_+ stand for the values on the horizon of Δ_L and Δ respectively. The condition $V(r_+) = 0$ yields

$$\Delta_L^+ = -\frac{\Delta_+^2}{g^2 r_+^2}\,. \tag{5.10}$$

Inserting this into (5.9), we finally obtain

$$\left(\frac{dr}{d\lambda}\right)^2\bigg|_{r_+} = -\frac{1}{g^2 r_+^2}\left(\Delta_+ E + \frac{2g^2}{r_+^2} a j_R\right)^2 \le 0\,. \tag{5.11}$$

If the previous quantity is strictly negative, it is obvious that the geodesics cannot reach the horizon, but bounce back. In the limiting case

$$E = -\frac{2g^2}{r_+^2 \Delta_+} a j_R\,, \tag{5.12}$$

the analysis needs more care. Writing the equation of motion (5.7) as $\dot{r}^2 + \mathrm{U}(r) = 0$, we see that it describes the classical one-dimensional motion of a particle in the potential $\mathrm{U}(r)$. If $\mathrm{U}'(r_+) \neq 0$, then r_+ is a turning point, and the geodesic bounces back without crossing the horizon. In contrast, if $\mathrm{U}'(r_+)$ vanishes, the particle needs an infinite time to reach r_+, i.e. the geodesic approaches asymptotically the horizon for $\lambda \to \infty$. But λ is just the affine parameter, and hence the geodesic ends there. Note that this result still holds true for spacelike geodesics ($m^2 < 0$).

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Hence, *no geodesic can cross the horizon and penetrate the black hole region in the spacetime* (2.2). This means, moreover, that the $r > r_+$ region of the manifold is *geodesically complete*, and contains no singularity.[4]

5.2 Charged geodesics

We have seen that geodesics in our manifold cannot cross the horizon. We turn now to the study of charged geodesics, and see if this property also holds for charged particles. The Hamiltonian follows from the minimal coupling with the U(1) potential,

$$H = \frac{1}{2} g^{\mu\nu} \left(p_\mu + QA_\mu\right) \left(p_\nu + QA_\nu\right), \tag{5.13}$$

where Q is the charge of the particle. The Hamilton-Jacobi equation can be separated again by the ansatz (5.2), and reads for charged geodesics

$$\begin{aligned} \left(\frac{dr}{d\lambda}\right)^2 &= \Delta_L E^2 - V\left(m^2 + \frac{4j^2}{r^2}\right) - \frac{4\Delta}{r^4} a j_R E - \frac{4g^2a^2}{r^6} j_R^2 - \\ &\quad -6\Delta QE - \frac{12g^2a}{r^2} j_R Q - 9\left(\frac{g^2a^2}{r^4} - \Delta^2\right) Q^2 . \end{aligned} \tag{5.14}$$

For $r = r_+$, the equation becomes

$$\left.\left(\frac{dr}{d\lambda}\right)^2\right|_{r_+} = -\frac{1}{g^2 r_+^2}\left(\Delta_+ E + \frac{2g^2}{r_+^2} a j_R + 3g^2 r_+^2 Q\right)^2 < 0 . \tag{5.15}$$

Being negative on the horizon, we can extend the previous conclusion to the propagation of charged particles: *charged geodesics cannot cross the horizon of the black holes under consideration.*

5.3 Scalar field propagation

We consider now a neutral, minimally coupled scalar field propagating in the background (2.2). The Klein-Gordon equation $\left(\nabla^2 - m^2\right)\Phi = 0$ reads

$$\begin{aligned} -\frac{1}{r^3}\frac{\partial}{\partial r}\left(r^3 V \frac{\partial \Phi}{\partial r}\right) &= -m^2\Phi - \frac{\Delta_L}{V}\frac{\partial^2\Phi}{\partial t^2} - \frac{4a\Delta}{r^4 V}\frac{\partial}{\partial t}\left(L_3\Phi\right) - \\ &\quad + \frac{4}{r^2}\left(L_1^2 + L_2^2 + L_3^2\right)\Phi + \frac{4g^2a^2}{r^6 V} L_3^2 \Phi . \end{aligned} \tag{5.16}$$

The variables can again be separated due to the symmetries of the problem. We use the ansatz

$$\Phi = F(r) e^{-iEt} D^j_{j_L, j_R} , \tag{5.17}$$

[4] As a consequence, the solutions under consideration are *not* black holes. However, allowing for a slight abuse of language, we shall continue to speak about "black holes" and "horizons" for convenience.

with $D^j_{j_L,j_R}$ the Wigner D-functions, which are simultaneous eigenfunctions of L_3, R_3, L^2 and R^2:

$$\begin{aligned}
R_3 D^j_{j_L,j_R} &= -ij_L D^j_{j_L,j_R}\,, \qquad L_3 D^j_{j_L,j_R} = ij_R D^j_{j_L,j_R}\,,\\
L^2 D^j_{j_L,j_R} &= R^2 D^j_{j_L,j_R} = -j(j+1) D^j_{j_L,j_R}\,.
\end{aligned} \tag{5.18}$$

We obtain then the radial wave equation,

$$-\frac{V}{r^3}\frac{d}{dr}\left(r^3 V \frac{dF}{dr}\right) = \left[\Delta_L E^2 - V\left(m^2 + \frac{4j(j+1)}{r^2}\right) - \frac{4a\Delta}{r^4}Ej_R - \frac{4g^2}{r^6}a^2 j_R^2\right] F\,. \tag{5.19}$$

To eliminate the first order derivative from (5.19), we introduce a tortoise-like coordinate

$$r_*(r) = \int \frac{dr}{r^3 V(r)}\,, \tag{5.20}$$

which is a smooth strictly decreasing function of the radial coordinate r in the outer region; its range is $r_* \in]0,+\infty[$, where $r = r_+$ corresponds to $r_* \to +\infty$ and the spatial infinity $r \to +\infty$ corresponds to $r_* = 0$. In terms of this new variable, the radial wave equation reads

$$\frac{d^2 F}{dr_*^2} + \mathrm{U}(r_*) F = 0\,, \tag{5.21}$$

where we have defined the potential function

$$\mathrm{U}(r_*) = r^6 \left[\Delta_L E^2 - V\left(m^2 + \frac{4j(j+1)}{r^2}\right) - \frac{4a\Delta}{r^4}Ej_R - \frac{4g^2}{r^6}a^2 j_R^2\right]\,. \tag{5.22}$$

We are now interested in the behaviour of equation (5.21) near the horizon (i.e. for $r_* \to \infty$). To this end, we first observe that the potential U converges for $r \to \infty$; let us denote its asymptotic value by U_0,

$$\mathrm{U}_0 \equiv \lim_{r_* \to \infty} \mathrm{U}(r_*) = -\frac{r_+^4}{g^2}\left(\Delta_+ E + \frac{2g^2 a j_R}{r_+^2}\right)^2\,. \tag{5.23}$$

Hence, performing a Taylor-Laurent expansion of $\mathrm{U}(r_*)$ near infinity, the coefficients of the positive powers vanish, and we are left with $\mathrm{U}(r_*) = \mathrm{U}_0 + \mathcal{O}(r_*^{-1})$. The behaviour near $r_* \to \infty$ of the solutions of the radial equation (5.21) is determined by the sign of U_0: if it is positive, we have oscillating solutions for $r \to r_+$, otherwise the solutions are exponentially depressed or diverging. From equation (5.23), we see that U_0 is always negative, and oscillating solutions are therefore not possible. This means that the net flux of particles through $r = r_+$ vanishes, and the total absorption cross section of the horizon is zero:

$$\sigma_{\mathrm{abs}} = 0\,. \tag{5.24}$$

This result is exact and holds for any frequency. In fact it generalizes our previous conclusions on geodesic motion, which is the high frequency, WKB approximation. As a consequence of the non-existence of near-horizon oscillating modes, there is no particle production and the Hawking temperature vanishes, $T_{BH} = 0$. Furthermore, as the absorption cross section $\sigma_{\rm abs}$ is a measure for the horizon area, (5.24) suggests that one should assign zero entropy to the black holes (2.2).

6. General solution with vector supermultiplets

The results obtained in the Einstein-Maxwell theory can be straightforwardly generalized to $\mathcal{N} = 2$, $D = 5$ gauged supergravity coupled to n vector supermultiplets. The bosonic part of the lagrangian is given by

$$e^{-1}\mathcal{L} = \frac{1}{2}R + g^2\mathcal{W} - \frac{1}{4}G_{IJ}F^I_{\mu\nu}F^{J\mu\nu} - \frac{1}{2}G_{IJ}\partial_\mu X^I\partial^\mu X^J + \frac{e^{-1}}{48}\epsilon^{\mu\nu\rho\sigma\lambda}C_{IJK}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda\,, \tag{6.1}$$

where $I = 0, \ldots, n$. The scalar potential reads

$$\mathcal{W}(X) = \mathcal{W}_I\mathcal{W}_J\left(6X^IX^J - \frac{9}{2}\mathcal{G}^{ij}\partial_iX^I\partial_jX^J\right). \tag{6.2}$$

Here $\mathcal{W}_I$ specify the appropriate linear combination of the vectors that comprise the graviphoton of the theory, $\mathcal{A}_\mu = \mathcal{W}_IA^I_\mu$. The X^I are functions of the n real scalar fields, and obey the condition

$$\mathcal{V} = \frac{1}{6}C_{IJK}X^IX^JX^K = 1\,. \tag{6.3}$$

The gauge and the scalar couplings are determined in terms of the homogeneous cubic polynomial $\mathcal{V}$ which defines a "very special geometry" [11]. They are given by

$$\begin{aligned} G_{IJ} &= -\frac{1}{2}\partial_I\partial_J\log\mathcal{V}\Big|_{\mathcal{V}=1}\,, \\ \mathcal{G}_{ij} &= \partial_iX^I\partial_jX^JG_{IJ}\Big|_{\mathcal{V}=1}\,, \end{aligned} \tag{6.4}$$

where ∂_i and ∂_I refer, respectively, to partial derivatives with respect to the scalar field ϕ^i and $X^I = X^I(\phi^i)$.

For Calabi-Yau compactifications of M-theory, $\mathcal{V}$ denotes the intersection form, and X^I and $X_I \equiv \frac{1}{6}C_{IJK}X^JX^K$ correspond to the size of the two- and four-cycles of the Calabi-Yau threefold respectively. Here C_{IJK} are the intersection numbers of the threefold. In the Calabi-Yau cases, n is given by the Hodge number $h_{(1,1)}$.

The general charged rotating supersymmetric solution of the theory (6.1) reads [8]

$$\begin{aligned} ds^2 &= -g^2r^2e^{2U}dt^2 - e^{-4U}\left(dt - \frac{\alpha}{r^2}\sin^2\theta d\phi + \frac{\alpha}{r^2}\cos^2\theta d\psi\right)^2 + e^{2U}\left(\frac{dr^2}{V(r)} + r^2d\Omega_3^2\right), \\ A^I_\phi &= -e^{-2U}X^I\frac{\alpha}{r^2}\sin^2\theta, \quad A^I_\psi = e^{-2U}X^I\frac{\alpha}{r^2}\cos^2\theta, \quad A^I_t = e^{-2U}X^I, \end{aligned} \tag{6.5}$$

where

$$V(r) = 1 + g^2 r^2 e^{6U} - \frac{g^2\alpha^2}{r^4}\,. \tag{6.6}$$

As a particular case, one obtains for the $STU = 1$ model[5] ($X^0 = S$, $X^1 = T$, $X^2 = U$)

$$e^{6U} = H_1 H_2 H_3\,, \qquad H_I = h_I + \frac{q_I}{r^2}\,, \quad I = 0,1,2, \tag{6.7}$$

and

$$X^I = e^{2U} H_I^{-1}\,. \tag{6.8}$$

Taking all charges to be equal, $q_I = q$, and $h_I = 1$ for $I = 0,1,2$, the solution of the $STU = 1$ model reduces to the Einstein-Maxwell solution (2.2) considered in the previous sections. Using Euler coordinates on the three-sphere and the vielbeins (2.12), the general metric (6.5) can be cast into the form

$$ds^2 = -g^2 r^2 e^{2U} dt^2 - e^{-4U}\left(dt + \frac{\alpha}{2r^2}\,\sigma^3\right)^2 + e^{2U}\left(\frac{dr^2}{V(r)} + r^2 d\Omega_3^2\right). \tag{6.9}$$

"Horizons" occur for $r = r_+$ with $V(r_+) = 0$, while the VLS is located at the zero r_{VLS} of the function $\Delta_L(r)$, defined by

$$\Delta_L(r) = 1 - \frac{\alpha^2}{r^6 e^{6U}}\,. \tag{6.10}$$

It is straightforward to show that $V(r) > g^2 r^2 e^{6U}\Delta_L(r)$, from which it follows that the VLS is always external to the horizon, $r_+ < r_{VLS}$, as in the Einstein-Maxwell case: there are always naked closed timelike curves.

6.1 Geodesic motion and scalar field propagation

To see if the general solution represents a black hole, we have to study its maximal extension. The Hamilton-Jacobi equation for the geodesic motion is

$$\begin{aligned} &-\frac{\Delta_L}{V} e^{4U}\left(\frac{\partial S}{\partial t}\right)^2 + V e^{-2U}\left(\frac{\partial S}{\partial r}\right)^2 - \frac{4\alpha}{r^4 V} e^{-2U}\frac{\partial S}{\partial t}\,(L_3 S) + \\ &+\frac{4}{r^2} e^{-2U}\left((L_1 S)^2 + (L_2 S)^2 + (L_3 S)^2\right) + \frac{4g^2\alpha^2}{r^6 V} e^{-2U}\,(L_3 S)^2 = -m^2\,. \end{aligned} \tag{6.11}$$

Again, the Hamilton-Jacobi equation can be completely separated using the ansatz (5.2). The orbital motion is the same as for the Einstein-Maxwell case, while the radial equation of motion becomes

$$\left(\frac{dr}{d\lambda}\right)^2 = e^{-4U}\left[e^{6U}\Delta_L E^2 - V\left(m^2 e^{2U} + \frac{4j^2}{r^2}\right) - \frac{4\alpha}{r^4} j_R E - \frac{4g^2\alpha^2}{r^6} j_R^2\right]. \tag{6.12}$$

[5] We apologize for using the same symbol for one of the moduli and the function appearing in the metric, but the meaning should be clear from the context.

To see if the geodesics can cross the horizon, we compute the r.h.s. of eq. (6.12) for $r = r_+$, with r_+ a zero of the function $V(r)$. This yields

$$\left(\frac{dr}{d\lambda}\right)^2\bigg|_{r_+} = -\frac{e^{-4\,\mathrm{U}(r_+)}}{g^2 r_+^2}\left(E + \frac{2g^2\alpha}{r_+^2} j_R\right)^2 \le 0\,. \tag{6.13}$$

This is negative, and applying the argument of section 5 we can conclude that no geodesic can cross the $r = r_+$ hypersurface. Hence, the region $r > r_+$ is geodesically complete and non-singular.

The Klein-Gordon equation for a neutral, minimally coupled scalar field of mass m propagating in the background of the general solution (6.9) reads

$$\begin{aligned}-\frac{e^{-2U}}{r^3}\frac{\partial}{\partial r}\left(r^3 V\frac{\partial\Phi}{\partial r}\right) &= -m^2\Phi - e^{4U}\frac{\Delta_L}{V}\frac{\partial^2\Phi}{\partial t^2} - e^{-2U}\frac{4\alpha}{r^4 V}\frac{\partial}{\partial t}(L_3\Phi) + \\ &+ \frac{4e^{-2U}}{r^2}\left(L_1^2 + L_2^2 + L_3^2\right)\Phi + \frac{4g^2\alpha^2}{r^6 V}e^{-2U}L_3^2\Phi\,. \end{aligned}\tag{6.14}$$

Making the ansatz (5.17), the variables separate, leaving the radial wave equation

$$-\frac{1}{r^3}\frac{d}{dr}\left(r^3 V\frac{dF}{dr}\right) = \left[e^{6U}\frac{\Delta_L}{V}E^2 - m^2 e^{2U} - \frac{4j(j+1)}{r^2} - \frac{4\alpha}{r^4 V}E j_R - \frac{4g^2\alpha^2}{r^6 V}j_R^2\right]F\,. \tag{6.15}$$

Using the Regge-Wheeler coordinate r_* defined in (5.20), we obtain the differential equation

$$\frac{d^2F}{dr_*^2} + P(r_*)F = 0\,, \tag{6.16}$$

describing a classical particle moving in the potential

$$P(r_*) = r^6\left[e^{6U}\Delta_L E^2 - V\left(m^2 e^{2U} + \frac{4j(j+1)}{r^2}\right) - \frac{4\alpha}{r^4}E j_R - \frac{4g^2\alpha^2}{r^6}j_R^2\right]. \tag{6.17}$$

For $r = r_+$ ($r_* \to \infty$), the potential converges to the finite value P_0,

$$P_0 \equiv \lim_{r_*\to\infty} P(r_*) = -\frac{r_+^4}{g^2}\left(E + \frac{2g^2\alpha}{r_+^2}j_R\right)^2\,, \tag{6.18}$$

which is always negative. Hence, as shown previously, the total flux across the $r = r_+$ hypersurfaces is zero, and the total absorption cross section vanishes, $\sigma_{\text{abs}} = 0$. These results suggest to assign zero temperature and entropy also to the general solution (6.9).

7. Lifting to type-IIB supergravity

Let us now focus on the $STU = 1$ model, with scalar potential (6.2) and gauge couplings G_{IJ} (6.4) given by [12][6]

$$\mathcal{W} = 2\left(\frac{1}{U} + \frac{1}{T} + TU\right),$$

[6]We assumed $h_I = 1$, yielding $\mathcal{W}_I = 1/3$ [8].

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$$G_{IJ} = \frac{1}{2}\begin{pmatrix} T^2U^2 & 0 & 0 \\ 0 & 1/T^2 & 0 \\ 0 & 0 & 1/U^2 \end{pmatrix}. \tag{7.1}$$

In [13] it was shown that this model can be obtained from type-IIB supergravity in ten dimensions by the Kaluza-Klein reduction ansatz

$$ds_{10}^2 = \sqrt{\tilde{\Delta}} ds_5^2 + \frac{1}{g^2\sqrt{\tilde{\Delta}}}\sum_{I=0}^{2}(X^I)^{-1}\left(d\mu_I^2 + \mu_I^2(d\phi_I + gA^I)^2\right), \tag{7.2}$$

where the three quantities μ_I are subject to the constraint $\sum_I \mu_I^2 = 1$. The standard metric on the unit five-sphere can be written in terms of these as

$$d\Omega_5^2 = \sum_I (d\mu_I^2 + \mu_I^2 d\phi_I^2). \tag{7.3}$$

The μ_I can be parametrized in terms of angles on a two-sphere, e.g. as

$$\mu_0 = \sin\Theta, \qquad \mu_1 = \cos\Theta\sin\Psi, \qquad \mu_2 = \cos\Theta\cos\Psi. \tag{7.4}$$

$\tilde{\Delta}$ is given by

$$\tilde{\Delta} = \sum_{I=0}^{2} X^I \mu_I^2. \tag{7.5}$$

The ansatz for the reduction of the 5-form field strength is $F_{(5)} = G_{(5)} + *G_{(5)}$, where [13]

$$\begin{aligned} G_{(5)} = {} & 2g\sum_I \left((X^I)^2\mu_I^2 - \tilde{\Delta}X^I\right)\epsilon_{(5)} - \frac{1}{2g}\sum_I (X^I)^{-1}\bar{*}dX^I \wedge d(\mu_I^2) + \\ & + \frac{1}{2g^2}\sum_I (X^I)^{-2} d(\mu_I^2) \wedge (d\phi_I + gA^I) \wedge \bar{*}dA^I, \end{aligned} \tag{7.6}$$

here $\epsilon_{(5)}$ is the volume form of the five-dimensional metric ds_5^2, and $\bar{*}$ denotes the Hodge dual with respect to the five-dimensional metric ds_5^2.

The other bosonic fields of the type-IIB theory are zero in this $\mathrm{U}(1)^3$ truncated reduction.

Using (7.2), we can lift our charged rotating supersymmetric solutions (6.9), with U given by (6.7), to ten dimensions, obtaining

$$\begin{aligned} ds_{10}^2 = {} & \sqrt{\tilde{\Delta}}\left[-g^2r^2e^{2U}dt^2 - e^{-4U}\left(dt + \frac{\alpha}{2r^2}\sigma^3\right)^2 + e^{2U}\left(\frac{dr^2}{V(r)} + r^2 d\Omega_3^2\right)\right] + \\ & + \frac{1}{g^2\sqrt{\tilde{\Delta}}}\sum_{I=0}^{2}(X^I)^{-1}\left(d\mu_I^2 + \mu_I^2(d\phi_I + gH_I^{-1}(dt + \frac{\alpha}{2r^2}\sigma^3))^2\right). \end{aligned} \tag{7.7}$$

(7.7) represents a D3-brane rotating both in directions transverse and longitudinal to the world volume.

Note that in five dimensions, the norm squared of the Killing vector ∂_γ is

$$(\partial_\gamma, \partial_\gamma)_{(5)} = e^{2U}\frac{r^2}{4}\Delta_L(r)\,, \tag{7.8}$$

where $\Delta_L(r)$ is given by eq. (6.10). As we said, for $r < r_{VLS}$ (7.8) becomes negative, so we have closed timelike curves. We would like to see whether these CTCs still occur in ten dimensions. A straightforward calculation shows that the norm squared of ∂_γ computed with the ten-dimensional metric (7.7) reads

$$(\partial_\gamma, \partial_\gamma)_{(10)} = \sqrt{\tilde{\Delta}}e^{2U}\frac{r^2}{4}\,, \tag{7.9}$$

which is always positive. Thus in the ten-dimensional metric ∂_γ is always spacelike, and the CTCs present in five dimensions disappear upon lifting. However, new CTCs show up. Consider e. g. the vector

$$v := -\frac{e^{-2U}\alpha g}{2r^2}\sum_{I=0}^{2}\partial_{\phi_I} + \partial_\gamma, \tag{7.10}$$

which has closed orbits, and norm

$$v^2 = e^{2U}\frac{r^2}{4}\Delta_L(r)\,, \tag{7.11}$$

that becomes negative for $r < r_{VLS}$. Thus in our case we have no resolution of causal anomalies in higher dimensions, in contrast to the BMPV black hole [6].

The naively computed Bekenstein-Hawking entropy of the rotating D3 brane (7.7) reads

$$S = \frac{V_3 V_5}{4g^5 G_{10}}\sqrt{e^{6\,\mathrm{U}(r_+)}r_+^6 - \alpha^2}\,, \tag{7.12}$$

where V_3 and V_5 denote the volume of the unit $\mathcal{S}^3$ and $\mathcal{S}^5$ respectively. (7.12) is clearly imaginary and thus makes no sense.

8. Final remarks

The solutions (2.2) and (6.5) that we studied in this paper are BPS states preserving half of the supersymmetry of $\mathcal{N} = 2$, $D = 5$ gauged supergravity. The fact that we obtained a vanishing Hawking temperature T_H due to the non-existence of near-horizon oscillating modes is in agreement with the supersymmetry considerations. Note that we obtained $T_H = 0$ also for solutions with a simple root r_+ of the function $V(r)$. One would expect for such solutions a bifurcated horizon, and a finite Hawking temperature. The resolution of this puzzle is related to the bad causal behaviour of the spacetimes under consideration.

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Another question to settle, is whether it is possible to construct and use such time machines. For example, one could take (for simplicity) the solution with $q = 0$ and $a = 0$, which is AdS_5, and try to add angular momentum by turning on a. This leads to CTCs outside the horizon. But this process, which is similar to accelerating a particle beyond the speed of light [5], should be impossible due to Hawking's chronology protection conjecture [21]. We expect furthermore, in analogy with the ungauged case [6], that these causality-violating solutions should be forbidden, because they would correspond to states which violate the unitarity bound of the dual $\mathcal{N} = 4$, $D = 4$ SYM theory. In such a way, the AdS/CFT correspondence would provide us with a nice implementation of the chronology protection conjecture in AdS spacetimes.

This effect already shows up for the uncharged nonextremal Kerr-AdS_5 black holes found in [22], where CTCs appear outside the horizon if the two rotation parameters are equal, and if the mass parameter M is sufficiently negative. From the dual CFT point of view, it is clear that these CTCs are related to loss of unitarity. The reason is that the classification of unitary representations of superconformal algebras typically implies inequalities on the conformal weights and R-charges. These inequalities thus yield lower bounds on the black hole mass M, and therefore the unitarity bound of the superconformal algebra is violated if M becomes too negative. It remains to be shown that this unitarity bound exactly coincides with the point in parameter space where CTCs develop.

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